

转移概率一般有界的Markov跳变系统有限时间保成本控制

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摘要: 针对系统信息不完全, 本文研究了转移概率一般有界的随机Markov跳变系统有限时间控制问题. 一般有界涵盖了部分未知、有界不确定和一般不确定3种不完全转移概率的情况. 首先, 研究该系统有限时间随机稳定问题, 引入自由连接权矩阵分离未知转移概率信息, 给出开环系统有限时间稳定的充分性判据; 其次, 借助Lyapunov方法和完全平方技术, 得到了实现系统有限时间保成本控制的充分条件, 给出了控制增益的设计方式以及性能指标的精确上界; 最后, 以太阳能锅炉控制系统为实例进行数值仿真, 说明本研究结果的有效性和实用性.

关键词: Markov跳变系统; 转移概率一般有界; 有限时间随机稳定; 保成本控制

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Finite-time guaranteed cost control for stochastic Markov jump system with generally bounded transition probabilities

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Abstract: Aiming at the issue of incomplete system information, this paper investigates the finite-time guaranteed cost control theory for stochastic Markov jump system with generally bounded transition probabilities, which is composed of three scenarios of incomplete transition probabilities: partially unknown, bounded uncertainties, and general uncertainties. Firstly, the finite-time stochastic stability problem is explored. By introducing a free connection weight matrix to decouple the unknown transition probability information, a sufficient criterion for the finite-time stability of the open-loop system is derived. Secondly, with the help of Lyapunov method and complete square technique, the sufficient conditions for realizing the finite-time guaranteed cost control of the system are obtained, and the design method of control gain and the exact upper bound of performance index are given. Finally, numerical simulations based on a solar boiler control system are conducted to demonstrate the validity and practicality of the research findings.

Key words: Markov jump system; generally bounded transition probabilities; finite-time stochastic stability; guaranteed cost control

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1 引言

Markov跳变系统是具有多个模态的随机系统, 它能够较好地刻画现实系统中外部环境变化而引发的子系统互联转变, 被广泛应用于网络故障与修复^[1]、

太阳能板发电量切换^[2]和交通限速调制^[3]等实际问题中. Markov跳变系统的运行受模态间转移概率的支配, 其信息的完全与否很大程度上影响着系统性能甚至稳定性^[4]. 然而, 现实中的转移概率很难被完全精

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确获取,例如,在太阳能锅炉系统中,Markov链被用于描述晴天、多云和阴天3种天气,其转移概率矩阵对应不同天气的切换.转移概率矩阵元素极难精确获得,存在不完全信息的情况,研究人员仅能根据气象统计数据获取其估计值及误差(称情形①转移概率有界不确定),或是一部分矩阵元素精确值(称情形②转移概率部分未知).Guo等^[5]考虑到这两种不完全信息的情况,提出了一般不确定转移概率(称情形③).该类型涵盖了情形①和②,虽较具一般性,但仍需获得一部分转移概率的估计值及误差.为进一步降低转移概率信息精确度的门槛,Li等^[6]提出一个新的且更为一般化的描述,称为转移概率一般有界.这一类型仅已知部分转移概率所在区间,转移概率有估计值及误差^[7-8],部分未知^[9]和一般不确定^[5]均可看作该类型的特例.这类系统的优势是仅需依赖粗略的转移概率信息,即可推导系统的稳定性条件及控制策略.

现有文献大多是基于Lyapunov渐进稳定性理论,研究无限时间系统稳定性.不同于无限时间系统,在实践中存在运行轨迹需要在特定的时间间隔内达到预定标准的有限时间系统,如导弹系统、通信网络系统和机器人操控系统等.这类系统关注短时间暂态稳定性,对应有限时间控制理论.Dorato^[10]首次提出了有限短时间稳定性的概念,考虑了一定时间间隔内的瞬态性能,适用于机器人控制^[11]、飞行器控制^[12]和通讯网络^[13]等实际问题,可见有限时间控制问题有很强的应用背景.线性矩阵不等式的提出将控制增益的求解转换为找到一组线性矩阵不等式可行解的问题,从而避开了求解Riccati方程的困难,推动了有限时间系统稳定性结果的实现^[14-19].此外,有些实际系统在满足有限时间稳定的同时,还要兼顾系统的性能指标,例如,电力系统期望电能消耗最小,同时不允许有大的暂态电压^[15],又如机械臂的角度位置被要求不超过一定的阈值^[20].这类控制问题一方面要求有限时间内系统是稳定的,另一方面需要最小化系统性能指标上界,即保成本控制.可见,有限时间保成本控制问题要同时满足以上两个方面,这比单一稳定性问题更为复杂,也契合实际需要^[14].

有鉴于此,本文针对转移概率一般有界的Markov跳变系统,研究有限时间保成本控制问题,得到使系统有限时间镇定且性能指标有上界的保成本控制策略.本文的创新点如下:

1) 在Markov跳变系统上,所研究的转移概率一般有界涵盖了转移概率部分未知^[9,21-22]、转移概率有误差^[7-8,23]和转移概率一般不确定^[5]3种情况,所得的研究结论更具普适性;

2) 在理论创新上,本文在Li等^[6]给出无限时间稳定性定理的基础上,得到了转移概率一般有界的Mar-

kov跳变系统有限时间随机稳定性定理,以应对现实系统对有限短时间稳定的需求.进一步,推导出有限保成本控制问题增益的显式表达和性能指标的精确上界;

3) 在仿真实例上,将文献[2,24]中离散Markov跳变系统在太阳能锅炉系统的应用延伸至转移概率一般有界的情况,实现了转移概率从完全信息到不完全信息的推进,所得保成本控制策略能够保证具有多类型不完全转移概率的离散Markov跳变系统有限时间稳定且性能指标有上界.

符号说明: \mathbb{M}^n : n 维的欧氏空间; $\mathbb{M}^{n \times m}$: 所有 $n \times m$ 阶实矩阵的集合; I : 单位矩阵; C^T : 矩阵 C 的转置; C^{-1} : 矩阵 C 的逆矩阵; $C > 0 (C \geq 0)$: 正定(半正定)对称矩阵; ϕ_{ab} : Kronecker函数,即若 $a = b$, $\phi_{ab} = 1$,若 $a \neq b$, $\phi_{ab} = 0$; $E\{\cdot\}$: 数学期望算子; $\|\cdot\|$: 向量的Euclidean范数或矩阵的谱范数; $\ell_{\max}(C) (\ell_{\min}(C))$: 矩阵 C 的最大(最小)特征值; $(\Omega, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$: 完备概率空间; $*$: 矩阵的对称项; N : 有限的正整数集合.

2 系统描述

考虑如下离散时间Markov跳变随机系统:

$$\begin{cases} x_{t+1} = \mathcal{A}(\theta_t)x_t + \mathcal{B}(\theta_t)u_t + \\ \quad [\mathcal{C}(\theta_t)x_t + \mathcal{D}(\theta_t)u_t]w_t, \\ x(0) = x_0, \end{cases} \quad (1)$$

其中: $x_t \in \mathbb{M}^n$ 是系统状态, $u_t \in \mathbb{M}^m$ 是控制输入, $\mathcal{A}(\theta_t), \mathcal{C}(\theta_t) \in \mathbb{M}^{n \times n}$, $\mathcal{B}(\theta_t), \mathcal{D}(\theta_t) \in \mathbb{M}^{n \times m}$ 是已知的系统参数矩阵. w_t 是一维标准维纳过程,满足 $E\{w_t\} = 0$ 和 $E\{w_a w_b\} = \phi_{ab}$. $\{\theta_t, t \geq 0\}$ 是Markov过程,取值在有限模态空间 $\mathcal{L} = \{1, 2, \dots, l\}$,其跳变转移概率矩阵 $\Pi = (p_{ij})$, $i, j \in \mathcal{L}$,满足

$$\Pr\{\theta_{t+1} = j | \theta_t = i\} = p_{ij}, \quad (2)$$

其中: $p_{ij} \geq 0$ 表示从时刻 t 模态 i 到 $t+1$ 时刻模态 j 的概率,满足 $\sum_{j=1}^l p_{ij} = 1$. w_t 与 θ_t 相互独立.本文假设Markov跳变过程的转移概率是一般有界的,如某个包含 n 个模态的系统具有如下转移概率矩阵:

$$\begin{bmatrix} [p_{11}, p_{u11}] & ? & \cdots & [p_{1n}, p_{un}] \\ ? & [p_{12}, p_{u12}] & ? & ? \\ ? & ? & ? & [p_{13}, p_{u13}] \\ [p_{14}, p_{u14}] & ? & \cdots & ? \end{bmatrix}, \quad (3)$$

其中: “?”表示未知转移概率; p_{ij}, p_{uij} 分别表示已知转移概率所在区间的上下界,定义集合 $\mathcal{L} = \mathcal{L}_k^i \cup \mathcal{L}_{uk}^i$,其中:

$$\mathcal{L}_k^i = \{j, p_{ij}, p_{uij} \text{ are known}\},$$

$$\mathcal{L}_{uk}^i = \{j, p_{uij}, p_{ij} \text{ are unknown}\}.$$

引入记号 $p_{ki} = \sum_{j \in \mathcal{L}_k^i} p_{ij}$, $i \in \mathcal{L}$,若 $\mathcal{L}_k^i = \emptyset$,则

$$\mathcal{L}_k^i = \{q_1^i, \dots, q_m^i\}, 1 \leq m \leq N,$$

其中: q_m^i 表示转移概率矩阵 Σ 中第 i 行第 m 个已知上下界的元素.

为表述方便, 假定 O 代指任意系数参数矩阵, 将 $\theta_t = i$ 时对应的 $O(i)$ 记为 O_i . 下面给出离散Markov跳变系统有限时间随机稳定的定义和相关引理.

定义 1^[18] 给定对称矩阵 $\mathcal{M} > 0$, 常数 $T > 0$ 和正整数 $c_1, c_2 (0 < c_1 < c_2)$, 若

$$\begin{aligned} E\{x_0^T \mathcal{M} x_0\} &\leq c_1 \Rightarrow \\ E\{x_t^T \mathcal{M} x_t\} &\leq c_2, \forall t \in [0, T], \end{aligned} \quad (4)$$

则系统(1)关于 $(c_1, c_2, T, \mathcal{M})$ 有限时间随机稳定.

注 1 本文基于转移概率一般有界的Markov跳变系统开展研究, 相应的研究结果可用于解决以下几种不完全转移概率:

- 1) 当 $\mathcal{L}_{uk}^i \neq \emptyset$, 转移概率所在区间的上下界满足 $p_{lij} = p_{uij}$ 时, 矩阵(3)为转移概率部分未知^[9, 25-26];
- 2) 当 $\mathcal{L}_{uk}^i \neq \emptyset$, 转移概率矩阵中元素的估计值为 $\frac{p_{lij} + p_{uij}}{2}$, 误差为 $\frac{p_{uij} - p_{lij}}{2}$ 时, 矩阵(3)为转移概率一般不确定^[5];
- 3) 当 $\mathcal{L}_{uk}^i = \emptyset$, 转移概率矩阵中元素的估计值为 $\frac{p_{lij} + p_{uij}}{2}$, 误差为 $\frac{p_{uij} - p_{lij}}{2}$ 时, 矩阵(3)为转移概率可容许不确定^[7-8, 24]. 因此, 本研究更具一般性.

3 有限时间稳定性问题

本节研究系统(1)有限时间随机稳定问题, 通过引入两个自由连接权矩阵, 分离未知转移概率信息, 给出有限时间随机稳定判据.

定理 1 对于未施加控制策略的离散时间Markov跳变系统(1), 若存在矩阵 $X_i > 0, W_i = W_i^T, H_{ij} = H_{ij}^T, i, j \in \mathcal{L}$, 正标量 α 满足下列不等式:

$$\mathcal{A}_i^T \rho_i \mathcal{A}_i + \mathcal{C}_i^T \rho_i \mathcal{C}_i - (\alpha + 1) \tilde{X}_i < 0, \quad (5)$$

$$\tilde{X}_j - W_i - H_{ij} \leq 0, j \in \mathcal{L}_k^i, \quad (6)$$

$$\tilde{X}_j - W_i \leq 0, j \in \mathcal{L}_{uk}^i, \quad (7)$$

$$\lambda_1 I < X_i < \lambda_2 I, \quad (8)$$

$$(\alpha + 1)^T c_1 \lambda_2 - c_2 \lambda_1 < 0, \quad (9)$$

其中:

$$\tilde{X}_j = \mathcal{M}^{\frac{1}{2}} X_j \mathcal{M}^{\frac{1}{2}}, \delta_{ij} = \frac{p_{uij} - p_{lij}}{2},$$

$$\rho_i = \sum_{j \in \mathcal{L}_k^i} p_{lij} \tilde{X}_j + \sum_{j \in \mathcal{L}_k^i} 2\delta_{ij} H_{ij} + (1 - \sum_{j \in \mathcal{L}_k^i} p_{lij}) W_i,$$

则离散时间Markov跳变系统(1)有限时间随机稳定.

证 令 $\theta_t = i, \theta_{t+1} = j$, 取随机Lyapunov函数

$$V(x_t, i) = x_t^T \tilde{X}_i x_t,$$

其中 $\tilde{X}_i = \mathcal{M}^{\frac{1}{2}} X_i \mathcal{M}^{\frac{1}{2}}$, 可得

$$\Delta V(x_t, i) =$$

$$E\{V(x_{t+1}, j) | \theta_t = i\} - V(x_t, i) =$$

$$E\{(\mathcal{A}_i x_t + \mathcal{C}_i x_t w_t)^T (\sum_{j=1}^l p_{ij} \tilde{X}_j) \times (\mathcal{A}_i x_t + \mathcal{C}_i x_t w_t)\} - x_t^T \tilde{X}_i x_t, \quad (10)$$

由于 w_t 独立于 x_t 和 u_t , 则

$$\begin{aligned} E\{x_t^T \mathcal{A}_i^T (\sum_{j=1}^l p_{ij} \tilde{X}_j) \mathcal{C}_i x_t w_t\} &= \\ E\{(\mathcal{C}_i x_t w_t)^T (\sum_{j=1}^l p_{ij} \tilde{X}_j) \mathcal{A}_i x_t\} &= 0. \end{aligned} \quad (11)$$

根据转移概率矩阵(3)中每行加和为1的属性, 对任意矩阵 $W_i = W_i^T$, 有 $(1 - \sum_{j \in \mathcal{L}} p_{ij}) W_i = 0$. 为方便书写, 令 $\omega_i = \omega_{1i} + \omega_{2i}$, 其中:

$$\begin{aligned} \omega_i &= \sum_{j \in \mathcal{L}} p_{ij} \tilde{X}_j + (1 - \sum_{j \in \mathcal{L}} p_{ij}) W_i, \\ \omega_{1i} &= \sum_{j \in \mathcal{L}_k^i} p_{ij} \tilde{X}_j + (1 - \sum_{j \in \mathcal{L}_k^i} p_{ij}) W_i, \\ \omega_{2i} &= \sum_{j \in \mathcal{L}_{uk}^i} p_{ij} \tilde{X}_j + (1 - \sum_{j \in \mathcal{L}_{uk}^i} p_{ij}) W_i, \end{aligned}$$

将 $\omega_i, \omega_{1i}, \omega_{2i}$ 和式(10)代入式(11), 可得

$$\begin{aligned} \Delta V(x_t, i) - \alpha V(x_t, i) &= \\ E\{x_t^T (\mathcal{A}_i^T \omega_i \mathcal{A}_i + \mathcal{C}_i^T \omega_i \mathcal{C}_i - (\alpha + 1) \tilde{X}_i) x_t\} &= \\ E\{x_t^T (\mathcal{A}_i^T \omega_{1i} \mathcal{A}_i + \mathcal{A}_i^T \omega_{2i} \mathcal{A}_i + \mathcal{C}_i^T \omega_{1i} \mathcal{C}_i + \mathcal{C}_i^T \omega_{2i} \mathcal{C}_i - (\alpha + 1) \tilde{X}_i) x_t\}, \end{aligned} \quad (12)$$

若式(7)成立, 则有

$$\mathcal{A}_i^T \omega_{2i} \mathcal{A}_i + \mathcal{C}_i^T \omega_{2i} \mathcal{C}_i \leq 0, \quad (13)$$

将式(13)和式(12)结合, 可得

$$\begin{aligned} \Delta V(x_t, i) - \alpha V(x_t, i) &\leq \\ E\{x_t^T (\mathcal{A}_i^T \omega_{1i} \mathcal{A}_i + \mathcal{C}_i^T \omega_{1i} \mathcal{C}_i - (\alpha + 1) \tilde{X}_i) x_t\}. \end{aligned} \quad (14)$$

根据转移概率矩阵(3)描述, 令 $p_{ij} = p_{\alpha ij} + p_{\beta ij}$, 其中:

$$\begin{aligned} p_{\alpha ij} &= \frac{p_{uij} + p_{lij}}{2}, p_{\beta ij} \in [-\delta_{ij}, \delta_{ij}], \\ \delta_{ij} &= \frac{p_{uij} - p_{lij}}{2}, \end{aligned}$$

式(14)可写为

$$\begin{aligned} \Delta V(x_t, i) &= \\ E\{x_t^T [\mathcal{A}_i^T (\sum_{j \in \mathcal{L}_k^i} (p_{\alpha ij} + p_{\beta ij}) \tilde{X}_j) \mathcal{A}_i + \mathcal{A}_i^T (1 - \sum_{j \in \mathcal{L}_k^i} p_{\alpha ij} - \sum_{j \in \mathcal{L}_k^i} p_{\beta ij}) W_i \mathcal{A}_i + \mathcal{C}_i^T (\sum_{j \in \mathcal{L}_k^i} (p_{\alpha ij} + p_{\beta ij}) \tilde{X}_j) \mathcal{C}_i + \mathcal{C}_i^T (1 - \sum_{j \in \mathcal{L}_k^i} p_{\alpha ij} - \sum_{j \in \mathcal{L}_k^i} p_{\beta ij}) W_i \mathcal{C}_i - \tilde{X}_i] x_t\}. \end{aligned} \quad (15)$$

根据式(6), 可得

$$\begin{aligned} & \sum_{j \in \mathcal{L}_k^i} (p_{\beta ij} + \delta_{ij})(\tilde{X}_j - W_i) \leq \\ & \sum_{j \in \mathcal{L}_k^i} (p_{\beta ij} + \delta_{ij})H_{ij} \leq \\ & \sum_{j \in \mathcal{L}_k^i} 2\delta_{ij}H_{ij}, \end{aligned} \quad (16)$$

将式(15)中的 $p_{\beta ij}$ 写为 $p_{\beta ij} + \delta_{ij} - \delta_{ij}$, 代式(16)入式(15), 化简整理可得

$$\begin{aligned} & \Delta V(x_t, i) - \alpha V(x_t, i) \leq \\ & E\{x_t^T [\mathcal{A}_i^T \rho_i \mathcal{A}_i + \mathcal{C}_i^T \rho_i \mathcal{C}_i - (\alpha + 1)\tilde{X}_i] x_t\}, \end{aligned} \quad (17)$$

结合式(5)–(7), 上式可推出

$$\Delta V(x_t, i) < \alpha V(x_t, i).$$

根据 $\Delta V(x_t, i)$ 的表达式, 有

$$\begin{cases} E\{V(x_1, \theta_1 = j) | x_0, \theta_0 = i\} - V(x_0, \theta_0) < \\ \alpha V(x_0, \theta_0), \\ E\{V(x_2, \theta_2 = j) | x_1, \theta_1 = i\} - V(x_1, \theta_1) < \\ \alpha V(x_1, \theta_1), \\ \vdots \\ E\{V(x_{t+1}, \theta_{t+1} = j) | x_t, \theta_t = i\} - V(x_t, \theta_t) < \\ \alpha V(x_t, \theta_t), \end{cases} \quad (18)$$

通过递归, 式(18)可写为

$$\begin{aligned} & E\{V(x_t, \theta_t = j)\} < \\ & (\alpha + 1)E\{V(x_{t-1}, \theta_{t-1})\} < \\ & (\alpha + 1)^2 E\{V(x_{t-2}, \theta_{t-2})\} < \\ & (\alpha + 1)^t E\{V(x_0, \theta_0)\}, \end{aligned}$$

根据 $E\{x_0^T \mathcal{M} x_0\} \leq c_1$, 得 $E\{V(x_t, i)\}$ 的上下界分别为

$$\begin{cases} E\{x_t^T \mathcal{M}^{\frac{1}{2}} X_i \mathcal{M}^{\frac{1}{2}} x_t\} \geq \ell_{\min}(X_i) E\{x_t^T \mathcal{M} x_t\}, \\ E\{x_t^T \mathcal{M}^{\frac{1}{2}} X_i \mathcal{M}^{\frac{1}{2}} x_t\} < \\ (\alpha + 1)^t \ell_{\max}(X_i) E\{x_0^T \mathcal{M} x_0\}. \end{cases} \quad (19)$$

根据式(8)–(9), 有

$$E\{x_t^T \mathcal{M} x_t\} < (\alpha + 1)^t \frac{\lambda_2 c_1}{\lambda_1}. \quad (20)$$

综上, 若式(5)–(9)成立, 可得 $E\{x_t^T \mathcal{M} x_t\} \leq c_2$, 未施加控制策略的系统(1)是有限时间随机稳定的。

证毕。

4 有限时间保成本控制

本节设计状态反馈控制器为:

$$u_t = K_i x_t, \quad (21)$$

用以镇定系统(1), 且使如下性能指标有上界:

$$J(x_0, i; u_t) = E\left\{\sum_{t=0}^T [x_t^T \mathcal{Q}_{ci} x_t] | \theta_0 = i\right\}, \quad (22)$$

其中: $\mathcal{Q}_{ci} = \mathcal{Q}_i + K_i^T \mathcal{R}_i K_i$, $\mathcal{Q}_i \geq 0$ 是状态权矩阵, $\mathcal{R}_i \geq 0$ 是控制权矩阵. 为方便书写, 将系统(1)简化为

$$\begin{cases} x_{t+1} = \mathcal{A}_{Ki} x_t + \mathcal{C}_{Ki} x_t w_t, \\ x(0) = x_0, \end{cases} \quad (23)$$

其中: $\mathcal{A}_{Ki} = \mathcal{A}_i + \mathcal{B}_i K_i$, $\mathcal{C}_{Ki} = \mathcal{C}_i + \mathcal{D}_i K_i$. 下面给出相关定义.

定义 2^[27] 对初始状态 $(x_0, i) \in (\mathbb{M}^n \times \mathcal{L})$, 若存在形如式(5)的状态反馈控制器使:

1) 离散时间Markov跳变系统(23)关于 $(c_1, c_2, T, \mathcal{M})$ 有限时间随机稳定;

2) $J(x_0, i; u_t) \leq J_s^*$, 则控制策略(21)称为有限时间状态反馈保成本控制器, J_s^* 为性能指标上界.

定理 2 给定矩阵 Z , 如果存在依赖于模态的矩阵 $X_i > 0$, $W_i = W_i^T$, $H_{ij} = H_{ij}^T$, $i, j \in \mathcal{L}$, 标量 $\eta, \alpha \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$, 满足下列不等式:

$$\begin{aligned} & \mathcal{A}_i^T \rho_i \mathcal{A}_i + \mathcal{C}_i^T \rho_i \mathcal{C}_i - (\alpha + 1)\tilde{X}_i + \\ & \mathcal{Q}_i - \eta_n Z^T \mathcal{T}_i - \eta \mathcal{T}_i^T Z + \eta^2 Z^T \mathcal{N}_i Z < 0, \end{aligned} \quad (24)$$

$$\tilde{X}_j - W_i - H_{ij} \leq 0, j \in \mathcal{L}_k^i, \quad (25)$$

$$\tilde{X}_j - W_i \leq 0, j \in \mathcal{L}_{uk}^i, \quad (26)$$

$$\lambda_1 I < X_i < \lambda_2 I, \quad (27)$$

$$(\alpha + 1)^T c_1 \lambda_2 - c_2 \lambda_1 < 0, \quad (28)$$

其中矩阵 Z 由0和单位阵或向量构成,

$$\tilde{X}_j = \mathcal{M}^{\frac{1}{2}} X_j \mathcal{M}^{\frac{1}{2}}, \delta_{ij} = \frac{p_{uij} - p_{lij}}{2},$$

$$\rho_i = \sum_{j \in \mathcal{L}_k^i} p_{lij} \tilde{X}_j + \sum_{j \in \mathcal{L}_k^i} 2\delta_{ij} H_{ij} + (1 - \sum_{j \in \mathcal{L}_k^i} p_{lij}) W_i,$$

则形如式(21)的有限时间保成本控制策略存在, 控制增益的显式表达为

$$K_i = -\mathcal{N}_i^{-1} \mathcal{T}_i,$$

其中: $\mathcal{N}_i = \mathcal{B}_i^T \rho_i \mathcal{B}_i + \mathcal{D}_i^T \rho_i \mathcal{D}_i + \mathcal{R}_i$, $\mathcal{T}_i = \mathcal{B}_i^T \rho_i \mathcal{A}_i + \mathcal{D}_i^T \rho_i \mathcal{C}_i$, 使Markov跳变系统(1)是关于 $(c_1, c_2, T, \mathcal{M})$ 有限时间随机稳定, 其中: 常数 $c_2 > c_1 > 0$, 矩阵 $\mathcal{M} > 0$, 同时, 性能指标函数(6)有上界, 即

$$J_s^* = \min\{[(\alpha + 1)^{T+1} + \alpha] \lambda_2 c_1\}. \quad (29)$$

证 令 $\theta_t = i$, $\theta_{t+1} = j$, 取Lyapunov函数为

$$V(x_t, i) = x_t^T \tilde{X}_i x_t,$$

其中 $\tilde{X}_i = \mathcal{M}^{\frac{1}{2}} X_i \mathcal{M}^{\frac{1}{2}}$, 可得

$$\begin{aligned} & \Delta V(x_t, i) = \\ & E\{V(x_{t+1}, j) | \theta_t = i\} - V(x_t, i) = \end{aligned}$$

$$\begin{aligned} & \mathbb{E}\{(\mathcal{A}_{K_i}x_t + \mathcal{C}_{K_i}x_t w_t)^\top (\sum_{j=1}^l p_{ij} \tilde{X}_j) \times \\ & (\mathcal{A}_{K_i}x_t + \mathcal{C}_{K_i}x_t w_t)\} - x_t^\top \tilde{X}_i x_t, \end{aligned} \quad (30)$$

由于 w_t 独立于 x_t 和 u_t , 则

$$\begin{aligned} & \mathbb{E}\{x_t^\top \mathcal{A}_{K_i}^\top (\sum_{j=1}^l p_{ij} \tilde{X}_j) \mathcal{C}_{K_i} x_t w_t\} = \\ & \mathbb{E}\{(\mathcal{C}_{K_i} x_t w_t)^\top (\sum_{j=1}^l p_{ij} \tilde{X}_j) \mathcal{A}_{K_i} x_t\} = 0. \end{aligned} \quad (31)$$

由定理1可知, 若式(25)和式(26)成立, 有

$$\begin{aligned} & \Delta V(x_t, i) - \alpha V(x_t, i) \leq \\ & \mathbb{E}\{x_t^\top [\mathcal{A}_i^\top \rho_i \mathcal{A}_i + \mathcal{C}_i^\top \rho_i \mathcal{C}_i - (\alpha + 1) \tilde{X}_i] x_t\}. \end{aligned} \quad (32)$$

由性能指标 $J(x_0, i; u_t) > 0$, 若

$$\begin{aligned} & J(x_0, i; u_t) + \mathbb{E}\{\sum_{t=0}^T [\Delta V(x_t, i)]\} - \\ & \alpha \mathbb{E}\{\sum_{t=0}^T [V(x_t, i)]\} < 0, \end{aligned} \quad (33)$$

则

$$\Delta V(x_t, i) < \alpha V(x_t, i). \quad (34)$$

通过递归, 可得

$$\mathbb{E}\{V(x_t, \theta_t = j)\} < (\alpha + 1)^t \mathbb{E}\{V(x_0, \theta_0)\}.$$

根据 $\mathbb{E}\{x_0^\top \mathcal{M} x_0\} \leq c_1$, 得 $\mathbb{E}\{V(x_t, i)\}$ 的上下界分别为

$$\begin{cases} \mathbb{E}\{x_t^\top \mathcal{M}^{\frac{1}{2}} X_i \mathcal{M}^{\frac{1}{2}} x_t\} \geq \ell_{\min}(X_i) \mathbb{E}\{x_t^\top \mathcal{M} x_t\}, \\ \mathbb{E}\{x_t^\top \mathcal{M}^{\frac{1}{2}} X_i \mathcal{M}^{\frac{1}{2}} x_t\} < \\ (\alpha + 1)^t \ell_{\max}(X_i) \mathbb{E}\{x_0^\top \mathcal{M} x_0\}. \end{cases} \quad (35)$$

根据式(35)和式(27), 有

$$\mathbb{E}\{x_t^\top \mathcal{M} x_t\} < (\alpha + 1)^t \frac{\lambda_2 c_1}{\lambda_1}. \quad (36)$$

结合式(28), 可得 $\mathbb{E}\{x_t^\top \mathcal{M} x_t\} \leq c_2$. 将式(33)展开, 可得

$$\begin{aligned} & J(x_0, i; u_t) + \mathbb{E}\{\sum_{t=0}^T [\Delta V(x_t, i)]\} - \\ & \alpha \mathbb{E}\{\sum_{t=0}^T [V(x_t, i)]\} < \\ & \mathbb{E}\{\sum_{t=0}^T [x_t^\top (\mathcal{A}_{K_i}^\top \rho_i \mathcal{A}_{K_i} + \mathcal{C}_{K_i}^\top \rho_i \mathcal{C}_{K_i} - \\ & (\alpha + 1) \tilde{X}_i + \mathcal{Q}_{ci}) x_t]\} = \\ & \mathbb{E}\{\sum_{t=0}^T [x_t^\top (\mathcal{A}_i^\top \rho_i \mathcal{A}_i + \mathcal{C}_i^\top \rho_i \mathcal{C}_i + \mathcal{Q}_i - \\ & (\alpha + 1) \tilde{X}_i) x_t + 2u_t^\top (\mathcal{B}_i^\top \rho_i \mathcal{A}_i + \mathcal{D}_i^\top \rho_i \mathcal{C}_i) x_t + \\ & u_t^\top (\mathcal{B}_i^\top \rho_i \mathcal{B}_i + \mathcal{C}_i^\top \rho_i \mathcal{C}_i + \mathcal{R}_i) u_t]\}, \end{aligned} \quad (37)$$

对式(37)右侧加和项使用完全平方法, 可得

$$\begin{aligned} & J(x_0, i; u_t) + \mathbb{E}\{\sum_{t=0}^T [\Delta V(x_t, i)]\} - \\ & \alpha \mathbb{E}\{\sum_{t=0}^T [V(x_t, i)]\} < \\ & \mathbb{E}\{\sum_{t=0}^T [x_t^\top (\mathcal{A}_i^\top \rho_i \mathcal{A}_i + \mathcal{C}_i^\top \rho_i \mathcal{C}_i + \mathcal{Q}_i + \\ & K_i^\top \mathcal{T}_i - (\alpha + 1) \tilde{X}_i) x_t + \\ & (u_t - K_i x_t)^\top \mathcal{N}_i (u_t - K_i x_t)]\}, \end{aligned} \quad (38)$$

其中控制增益 $K_i = -\mathcal{N}_i^{-1} \mathcal{T}_i$. 可得, 若

$$\begin{aligned} & \mathcal{A}_i^\top \rho_i \mathcal{A}_i + \mathcal{C}_i^\top \rho_i \mathcal{C}_i + \mathcal{Q}_i + \\ & K_i^\top \mathcal{T} - (\alpha + 1) \tilde{X}_i < 0, \end{aligned} \quad (39)$$

同时式(25)–(28)成立, 则系统(1)有限时间随机稳定. 注意到, 式(39)中的项 $-\mathcal{T}_i^\top \mathcal{N}_i^{-1} \mathcal{T}_i$ 是负定且非线性的, 这使该式无法用MATLAB中LMI工具箱进行求解, 为了消除这一项, 构造由0和单位阵或向量组成的矩阵 Z , 则对任意标量序列 η , 有

$$0 \leq (\mathcal{T}_i^\top - \eta Z^\top \mathcal{N}_i)^\top \mathcal{N}_i^{-1} (\mathcal{T}_i - \eta \mathcal{N}_i Z), \quad (40)$$

式(40)可以重写为

$$-\mathcal{T}_i^\top \mathcal{N}_i^{-1} \mathcal{T}_i \leq -\eta Z^\top \mathcal{T}_i - \eta \mathcal{T}_i^\top Z + \eta^2 Z^\top \mathcal{N}_i Z. \quad (41)$$

将式(41)代入式(39), 可得若式(24)–(28)成立, 系统(24)有限时间随机稳定. 同时, 性能指标有上界

$$\begin{aligned} & J(x_0, i; u_t) < \alpha \mathbb{E}\{\sum_{t=0}^T [V(x_t, i)]\} + V(x_0, \theta_0) = \\ & \alpha \mathbb{E}\{\sum_{t=0}^T (\alpha + 1)^t [V(x_0, 0)]\} + V(x_0, \theta_0) = \\ & \mathbb{E}\{((\alpha + 1)^{T+1} + \alpha) V(x_0, 0)\} = \\ & ((\alpha + 1)^{T+1} + \alpha) \lambda_2 c_1, \end{aligned} \quad (42)$$

则性能指标函数(22)有上界, 上界值可以通过求解如下优化问题获得

$$\begin{aligned} & \min [((\alpha + 1)^{T+1} + \alpha) \lambda_2 c_1], \\ & \text{s.t. (24)–(28)}. \end{aligned} \quad (43)$$

证毕.

注2 定理2将系统(23)有限时间保成本控制问题转化为求解满足一组线性矩阵不等式的优化问题, 由式(24)–(28)可知, 优化问题的求解与标量 $\eta, \alpha > 0$ 有关, 参考文献[27], 使用优化算法解决问题(43), 算法的伪代码如算法1(见表1)所示.

5 仿真实例

为验证以上结果的有效性和实用性, 本节将文献[2, 24]中太阳能锅炉控制系统仿真延伸至不完全信息的情况. 具体描述为: 太阳能锅炉系统由一组可调节的反射镜组成, 能够通过锅炉塔中流动的水使太阳光

聚焦. 当天天气决定太阳能转移到锅炉的功率, 即晴天、多云还是阴天. 天气晴朗时, 锅炉会获得更多的太阳能, 此时比多云和阴天条件下流量更大, 显然, 每种天气下的动力学过程不同. 根据经验, 大气条件可模拟为一个具有3种状态的Markov链: ① 晴朗, ② 多云, ③ 阴天. 研究员通常根据历史数据对3种天气状态的发生概率进行估计, 囿于各种内外部因素的影响, 概率估计可能存在以下情况: ① 数据缺失, ② 得到估计值和误差, ③ 仅能得到估计值的大致区间. 本文所研究的转移概率一般有界的Markov跳变系统将以上3种情况涵盖在内, 所得理论研究结论可用于解决这一实例在不完全信息下的有限时间保成本控制问题. 系统参数被赋值如下:

模态1: 晴天

$$\bar{A}_1 = \begin{bmatrix} 2 & -0.42 \\ 0.81 & -0.81 \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\bar{C}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathcal{D}_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$\mathcal{Q}_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \mathcal{R}_1 = 0.01.$$

模态2: 多云

$$\bar{A}_2 = \begin{bmatrix} 0.1 & -0.31 \\ -0.58 & 1 \end{bmatrix}, \mathcal{B}_2 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix},$$

$$\bar{C}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathcal{D}_2 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},$$

$$\mathcal{Q}_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \mathcal{R}_2 = 0.01.$$

模态3: 阴天

$$\bar{A}_3 = \begin{bmatrix} 0 & -0.31 \\ 0.1 & 0.71 \end{bmatrix}, \mathcal{B}_{13} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix},$$

$$\bar{C}_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathcal{D}_3 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$\mathcal{Q}_3 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \mathcal{R}_3 = 0.01.$$

3种天气状态间的转移概率矩阵如下:

$$\begin{bmatrix} [0.05 & 0.15] & ? & [0.2 & 0.4] \\ [0.4 & 0.6] & ? & ? \\ [0.4 & 0.6] & ? & ? \end{bmatrix}. \quad (44)$$

给定初值 $x_0 = \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}$, $\mathcal{M} = I$, $T = 10$, $\hat{\alpha} = 0.5$, 步长 $d_\alpha = 0.02$, $d_\eta = 0.02$, 借助MATLAB(2023)运行优化算法1, 可得性能指标上界为 $J_s^* = 0.0013$. 同时, 状态反馈控制增益为

$K_1 = [-1.3192 \ 0.0793]$, $K_2 = [0.3048 \ -0.6956]$, $K_3 = [-0.0858 \ -0.7942]$, 施加控制策略前后, 闭环系统(24)的 $E\{x_t^T \mathcal{M}x_t\}$ 分别如图1-2所示, 图1说明在有限时间 $T = 10$ s内, 闭环系统(24)的状态轨迹不在预定的界限内, 即 $E\{x_t^T \mathcal{M}x_t\} > c_2$, 则未施加控制策略时系统有限时间不稳定, 图2说明施加控制策略后系统状态轨迹在预定的界限内, 即满足 $E\{x_0^T \times \mathcal{M}x_0\} \leq c_1$, $E\{x_t^T \mathcal{M}x_t\} \leq c_2$, 系统(23)有限时间稳定.

表 1 优化算法

Table 1 Optimization algorithm

Input: c_1, c_2, T, \mathcal{M} 和估计的 $\hat{\alpha}, \bar{\eta}, \eta, \alpha$ 和 η 的步长 d_α, d_η .
Output: U_p, V_q 和 $W(p, q)$
 进行系统初始化, 令 $p = 1, q = 1$, 取 $\alpha^p = 0, \eta^q = \eta$.
if $\alpha^p < \hat{\alpha}$:
 While $\eta^q > \bar{\eta}$,
 if (α^p, η^q) 使式(24)–(28)有可行解, 求解优化问题(43), 进而, 将 (α^p, η^q) 存入 (U_p, V_q) , 将性能指标上界存入 $W(p, q)$.
 else
 令 $\alpha^p = \alpha^p + d_\alpha, p = p + 1$, 取 $q = 1, \eta^q = \eta$.
 end
else: 停止.

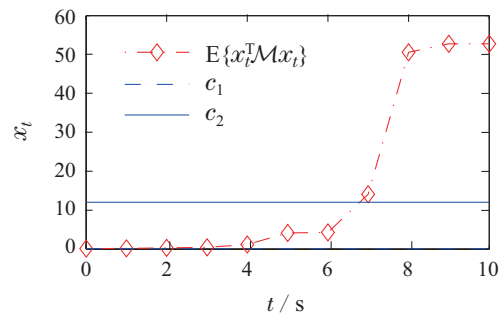


图1 开环系统(23)的 $E\{x_t^T \mathcal{M}x_t\}$

Fig. 1 The $E\{x_t^T \mathcal{M}x_t\}$ of the open-loop system (23)

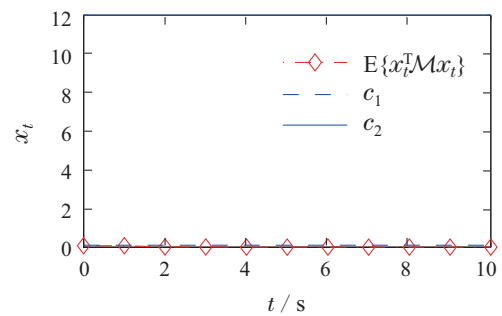


图2 闭环系统(23)的 $E\{x_t^T \mathcal{M}x_t\}$

Fig. 2 The $E\{x_t^T \mathcal{M}x_t\}$ of the close loop system (23)

为了说明定理2的一般性, 进一步补充转移概率矩阵(44)的信息, 将其变换成转移概率有界不确定的类型, 具体如下:

令 $\mathcal{L}_{uk}^i = \emptyset$, $p_{\alpha ij} = \frac{p_{lij} + p_{uij}}{2}$ 和 $p_{\beta ij} \in [-\delta_{ij}, \delta_{ij}]$, $\delta_{ij} = \frac{p_{uij} - p_{lij}}{2}$, 转移概率矩阵为

$$\begin{bmatrix} 0.1 + p_{\beta 11} & 0.5 + p_{\beta 12} & 0.3 + p_{\beta 13} \\ 0.5 + p_{\beta 21} & 0.2 + p_{\beta 22} & 0.1 + p_{\beta 23} \\ 0.5 + p_{\beta 31} & 0.4 + p_{\beta 32} & 0.1 + p_{\beta 33} \end{bmatrix},$$

其中: $\delta_{12} = 0.1$, $\delta_{22} = 0.1$, $\delta_{23} = \delta_{33} = 0.05$, $\delta_{32} = 0.2$, 使用定理2, 运行优化算法, 可得性能指标上界为 $J_s^* = 1.4321$, 状态反馈控制增益为:

$$K_1 = [-0.3315 \quad 0.0870], K_2 = [0.3681 \quad -0.8314], K_3 = [-0.2214 \quad -1.0892].$$

图3展示了施加控制策略后闭环系统状态轨迹, 由图3可知, 转移概率有界不确定的Markov跳变系统有限时间稳定.

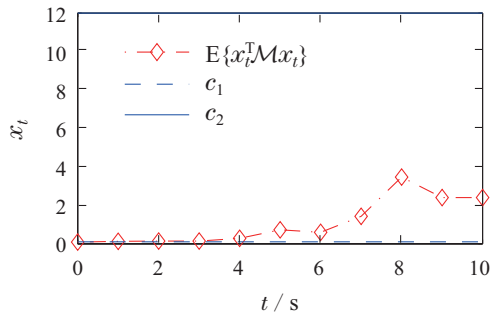


图3 闭环系统(23)的 $E\{x_t^T M x_t\}$

Fig. 3 The $E\{x_t^T M x_t\}$ of the close loop system (23)

6 结论

本文研究了转移概率一般有界的离散Markov跳变系统的有限时间保成本控制问题. 首先, 利用自由连接权矩阵分离未知转移概率信息, 使用Lyapunov函数方法得到了有限时间随机稳定判据. 随后, 结合完全平方技术推导出有限时间保成本控制增益的显式表达, 设计状态反馈控制器保证闭环系统有限时间稳定和性能指标有上界. 最后, 通过太阳能锅炉系统的仿真实例说明了在前文所得的控制策略下, 系统可以保持有限时间随机稳定. 对比现有研究, 本文探讨的转移概率一般有界的Markov跳变系统, 囊括了多类型不完全转移概率情况, 研究结果更具普适性.

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