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输入受限的核燃料装卸机边界振动控制

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摘要: 本文研究输入受限的核燃料装卸机的振动控制及位置控制. 为完成预期运行任务, 核燃料装卸机夹紧燃料棒在水下沿导轨移动. 本文考虑外部干扰的影响并分析燃料棒振动与装卸机运动间的耦合作用, 建立了非齐次高阶刚柔耦合分布参数模型. 运用反步法, 设计边界控制方案来运送燃料棒至给定位置, 并同时约束燃料棒的侧向及横向振动. 构造双曲正切函数以抑制外部干扰的影响. 引进辅助系统及Nussbaum函数来处理输入饱和问题. Lyapunov稳定性分析和仿真实验均证明了本文所提边界控制方案可以确保核燃料装卸机系统一致有界稳定.

关键词: 核燃料装卸机; 分布参数控制系统; 振动控制; 边界控制; 输入限制

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Boundary vibration control for a nuclear refueling machine with input constraint

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Abstract: This paper investigates the vibration and position control of a nuclear refueling machine with input constraints. To accomplish the intended operational tasks, the refueling machine grips fuel rods and moves underwater along guide rails. Taking into account the effect of external disturbances and analyzing the coupling between the vibration of the fuel rods and the motion of the refueling machine, a nonhomogeneous high-order rigid-flexible coupled distributed parameter model is developed. The backstepping method is employed to design a boundary control scheme aimed at transporting fuel rods to specified positions while simultaneously constraining their lateral and transverse vibrations. A hyperbolic tangent function is constructed to offset the influence of external disturbances. Auxiliary systems and a Nussbaum function are introduced to address the problem of input saturation. Both Lyapunov stability analysis and simulation experiments demonstrate that the boundary control scheme proposed in this paper can uniformly boundedly stabilize the nuclear refueling machine system.

Key words: nuclear refueling machine; distributed parameter control system; vibration control; boundary control; input constraint

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1 引言

随着人类社会对能源需求的快速增长, 核电作为清洁能源获得世界各国极大重视. 我国作为世界能源消耗最大的国家, 近些年来国家大力推动核电相关产业升级, 国产核电技术也取得长足进步. 核燃料棒作为核反应堆关键组件, 其性能直接决定核电站的安全可靠性. 核燃料装卸机在核电领域起着至关重要的作用, 其在水下对核燃料棒进行吊装及运输作业以完成检修和更换任务.

在复杂的水下环境中, 核燃料装卸机沿着导轨将燃料棒快而准地运送至目标位置[1–2]. 然而, 外部干扰和惯性力矩都会导致细长结构的燃料棒振动. 由于燃料棒的阻尼较小, 其振动很难在短时间内自然衰减[3]. 这些不理想的连续过激振动会影响燃料棒的使用寿命, 更严重的会导致生产事故. 因此, 本文将研究核燃料装卸机的振动控制及位置控制.

核燃料装卸机沿导轨做直线运输的同时其所携带的燃料棒存在弹性形变运动, 二者之间存在强烈刚柔

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耦合作用. 在实际工程中, 核燃料装卸机系统又不可避免的受外部干扰的影响. 因此, 核燃料装卸机系统是一个非齐次的刚柔耦合分布参数系统, 其动力学方程为耦合的偏微分–常微分方程[4–6]. 由于边界控制实现仅需获取系统边界状态信息且直接作用于系统边界处, 其已被用于控制许多不同类型的分布参数系统如柔性机械臂[7–8]、海洋立管[9–10]、扑翼无人机[11–12]等. 文献[13]解决了带输出约束的柔性结构系统的边界控制设计问题, 提出了一种新的障碍函数来满足期望约束条件; 文献[14]在鱼形机器人的鱼尾处安装边界控制器以完成复杂的躯体震荡运动; 文献[15]提出了迭代学习边界控制以抵消柔性旋转机械臂的弹性形变并同时实现姿态调节. 已有的边界控制方案大多用于控制柔性结构系统, 其不能直接用于镇定本文研究的非齐次刚柔耦合的核燃料装卸机系统. 此外, 输入饱和和干扰抑制为两个重要的控制工程问题, 忽略它们的影响会导致被控系统不稳定[16–17]. 文献[18]采用符号函数作为输入饱和模型, 构造辅助输入信号来补偿控制系统. 然而, 符号函数频繁切换状态会引起震颤问题. 本文采用光滑的双曲正切函数作为饱和模型, 并设计辅助系统来解决输入饱和问题; 文献[19]设计了结构复杂的干扰观测器来估计外部干扰的值, 并以负反馈的形式抵消外部干扰的影响; 文献[20]综合应用外部干扰的精确上界值及符号函数来实现干扰抑制. 本文引入双曲正切函数来抑制外部干扰, 其结构简单且实现不需要外部干扰的具体信息. 因此, 本文所设计的干扰抑制技术具有较好的可实现性和鲁棒性.

运用哈密顿原理, 本文将受外部干扰影响的核燃料装卸机系统建模为非齐次刚柔耦合分布参数系统. 综合运用反步法和边界控制技术, 构造了两个作用于运输小车及导轨上的边界控制律来抑制燃料棒的横向及侧向振动并同时实现位置跟踪. 引进双曲正切函数来抵消外部干扰. 设计辅助系统和Nussbaum函数来解决输入饱和问题. 对核燃料装卸机系统的一致有界稳定性进行了严格的Lyapunov分析. 数值仿真结果验证了所给控制方案的有效性.

2　动力学分析和预备知识

2.1　动力学分析

图1给出了沿导轨移动的核燃料装卸机物理示意图. 为了将燃料棒运输至期望位置, 小车和导轨分别移动至给定位置p_d及q_d. 根据柔性梁理论, 本文将细长的燃料棒视为欧拉–伯努利梁. $u(x,t)$和$v(x,t)$分别表示燃料棒的侧向和横向弹性形变量, $q(t)$和$p(t)$分别为导轨和小车的位置, EI为燃料棒的弯曲刚度, l为燃料棒的长度, d为燃料棒的直径, ρ为燃料棒的单位长度的质量, m_b为导轨的质量, m_r为小车的质量.

注 1　本文采用如下简写: $\dfrac{\partial(*)}{\partial x}=(*)'$, $\dfrac{\partial(*)}{\partial t}=\dot{(*)}$.

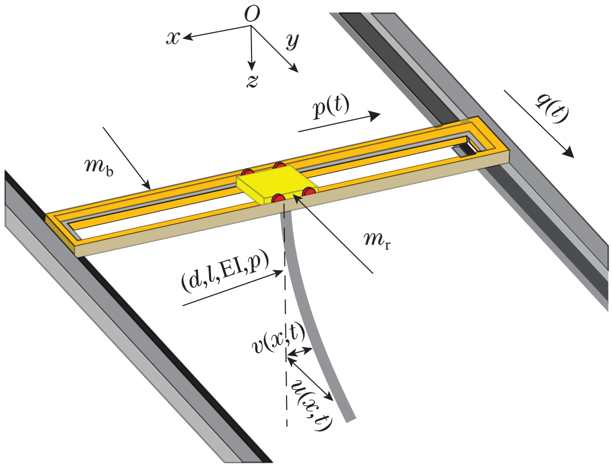

图 1　移动的核燃料装卸机

Fig. 1　A moving nuclear refueling machine

核燃料装卸机系统的动能为[21]

$$K(t)=\frac{\rho}{2}\int_0^l[\dot u(x,t)+\dot q(t)]^2\mathrm{d}x+\frac{\rho}{2}\int_0^l[\dot v(x,t)+\dot p(t)]^2\mathrm{d}x+\frac{m_\mathrm{b}+m_\mathrm{r}}{2}\dot q^2(t)+\frac{m_\mathrm{r}}{2}\dot p^2(t).\tag{1}$$

燃料棒弯曲形变导致的势能为[21]

$$P(t)=\frac{\mathrm{EI}}{2}\int_0^l[u''(x,t)]^2\mathrm{d}x+\frac{\mathrm{EI}}{2}\int_0^l[v''(x,t)]^2\mathrm{d}x.\tag{2}$$

非保守力做的功为

$$\begin{aligned}&\delta W(t)=\\&-c\int_0^l[\dot u(x,t)+\dot q(t)]\delta[u(x,t)+q(t)]\mathrm{d}x-\\&c\int_0^l[\dot v(x,t)+\dot p(t)]\delta[v(x,t)+p(t)]\mathrm{d}x+\\&\int_0^l F_\mathrm{q}(x,t)\delta[u(x,t)+q(t)]\mathrm{d}x+\\&\int_0^l F_\mathrm{p}(x,t)\delta[v(x,t)+p(t)]\mathrm{d}x+\\&u_\mathrm{q}(t)\delta q(t)+u_\mathrm{p}(t)\delta p(t)+\\&d_\mathrm{q}(t)\delta q(t)+d_\mathrm{p}(t)\delta p(t),\end{aligned}\tag{3}$$

其中: $F_\mathrm{q}(x,t)$和$F_\mathrm{p}(x,t)$分别是侧向和横向分布式干扰, $d_\mathrm{q}(t)$和$d_\mathrm{p}(t)$是边界干扰, $u_\mathrm{q}(t)$和$u_\mathrm{p}(t)$是待设计的边界控制律.

将系统动能、势能及外力做的功代入Hamilton原理[22] $\int_{t_1}^{t_2}[\delta K(t)-\delta P(t)+\delta W(t)]\mathrm{d}t=0$, 运用变分原理可得核燃料装卸机系统的控制方程为

$$\begin{aligned}&\rho\ddot u(x,t)+\mathrm{EI}u''''(x,t)+c\dot u(x,t)+\\&\rho\ddot q(t)+c\dot q(t)=F_\mathrm{q}(x,t),\end{aligned}\tag{4}$$

$$\rho\ddot{v}(x,t)+\mathrm{EI}v''''(x,t)+c\dot{v}(x,t)+ \rho\ddot{p}(t)+c\dot{p}(t)=F_{\mathrm{p}}(x,t), \tag{5}$$

边界条件为

$$u(0,t)=u'(0,t)=u''(l,t)=u'''(l,t)=0, \tag{6}$$

$$v(0,t)=v'(0,t)=v''(l,t)=v'''(l,t)=0, \tag{7}$$

$$(m_{\mathrm{b}}+m_{\mathrm{r}})\ddot{q}(t)=u_{\mathrm{q}}(t)+d_{\mathrm{q}}(t)-\mathrm{EI}u'''(0,t), \tag{8}$$

$$m_{\mathrm{r}}\ddot{p}(t)=u_{\mathrm{p}}(t)+d_{\mathrm{p}}(t)-\mathrm{EI}v'''(0,t). \tag{9}$$

2.2 预备知识

为了后文分析方便, 引进下列引理及假设.

引理 1[23] 对于任意的$w_1(x,t), w_2(x,t)\in\mathbb{R}$, 有

$$|w_1(x,t)w_2(x,t)|\leqslant\frac{1}{\psi}w_1^2(x,t)+\psi w_2^2(x,t),$$

其中$\psi\in\mathbb{R}^+$.

引理 2[24] 对于一阶连续可微函数$w(x,t)$, 有

$$w^2(x,t)\leqslant w^2(0,t)+l\int_0^l[w'(x,t)]^2\mathrm{d}x.$$

引理 3[25] 对于任意$x\in\mathbb{R}$, 存在正的常数c使得下列不等式成立:

$$0\leqslant|x|-x\tanh(\frac{x}{c})\leqslant\mu c,\ \mu=0.2785.$$

引理 4[25] 假设$V(\cdot)$及$\chi(\cdot)$均为$[0,t_{\mathrm{f}})$上的光滑函数, $N(\chi)$为$[0,t_{\mathrm{f}})$上的光滑Nussbaum函数, 则下列不等式成立:

$$V\leqslant c_0+\mathrm{e}^{-c_1t}\int_0^t(g(\tau)N(\chi)-1)\dot{\chi}\mathrm{e}^{c_1\tau}\mathrm{d}\tau,$$

其中: $c_1>0$, c_0为常数, $g(\cdot)$为在不包含0的闭区间取值的时变参数. 此外, $V(t)$, χ, $\int_0^t(g(\tau)N(\chi)-1)\dot{\chi}\mathrm{e}^{c_1\tau}\mathrm{d}\tau$在$[0,t_{\mathrm{f}})$有界.

假设 1 本文假设外部干扰有界, 即存在常数$F_1, F_2, M_1, M_2>0$, 使得

$$\begin{cases}F_{\mathrm{q}}(x,t)\leqslant F_1,\\ F_{\mathrm{p}}(x,t)\leqslant F_2,\\ d_{\mathrm{q}}(t)\leqslant M_1,\\ d_{\mathrm{p}}(t)\leqslant M_2.\end{cases}$$

3 边界控制

考虑外部干扰和输入饱和的影响, 本文研究移动的核燃料装卸机系统. 运用反步法, 设计边界控制来实现下列控制目标: 1) 抑制燃料棒的横向及侧向振动; 2) 驱动小车及导轨至期望位置; 3) 抵消外部干扰的影响; 4) 处理输入饱和问题.

针对带输入饱和的核燃料装卸机系统, 选取饱和模型如下[25]:

$$\begin{cases}u_{\mathrm{q}}(t)=\\ u(v_{\mathrm{q}}(t))=u_1\tanh(\dfrac{v_{\mathrm{q}}(t)}{u_1})=\\ u_1\dfrac{\mathrm{e}^{\frac{v_{\mathrm{q}}(t)}{u_1}}-\mathrm{e}^{\frac{-v_{\mathrm{q}}(t)}{u_1}}}{\mathrm{e}^{\frac{v_{\mathrm{q}}(t)}{u_1}}+\mathrm{e}^{\frac{-v_{\mathrm{q}}(t)}{u_1}}},\\ u_{\mathrm{p}}(t)=\\ u(v_{\mathrm{p}}(t))=u_2\tanh(\dfrac{v_{\mathrm{p}}(t)}{u_2})=\\ u_2\dfrac{\mathrm{e}^{\frac{v_{\mathrm{p}}(t)}{u_2}}-\mathrm{e}^{\frac{-v_{\mathrm{p}}(t)}{u_2}}}{\mathrm{e}^{\frac{v_{\mathrm{p}}(t)}{u_2}}+\mathrm{e}^{\frac{-v_{\mathrm{p}}(t)}{u_2}}},\end{cases} \tag{10}$$

其中: $u_1, u_2>0$为输入的上界; $v_{\mathrm{q}}(t)$和$v_{\mathrm{p}}(t)$为待设计的控制律.

为后文使用反步法设计控制律, 构造下列坐标变换:

$$\begin{cases}y_1=x_{1\mathrm{y}}=e_{\mathrm{q}}(t),\\ y_2=x_{2\mathrm{y}}-\tau_{1\mathrm{y}}=\dot{q}(t)-\tau_{1\mathrm{y}},\\ y_3=u(v_{\mathrm{q}}(t))-\tau_{2\mathrm{y}},\\ z_1=x_{1\mathrm{z}}-e_{\mathrm{p}}(t),\\ z_2=x_{2\mathrm{z}}-\tau_{1\mathrm{z}}=\dot{p}(t)-\tau_{1\mathrm{z}},\\ z_3=u(v_{\mathrm{p}}(t))-\tau_{2\mathrm{z}},\end{cases} \tag{11}$$

其中: $e_{\mathrm{q}}(t)=q(t)-q_{\mathrm{d}}$; $e_{\mathrm{p}}(t)=p(t)-p_{\mathrm{d}}$; $\tau_{1\mathrm{y}}, \tau_{2\mathrm{y}}, \tau_{1\mathrm{z}}$和$\tau_{2\mathrm{z}}$为虚拟控制.

由式(4)–(9)描述的原始动力学模型, 通过下列3步分析来设计边界控制律.

步骤 1 构造Lyapunov候选函数如下:

$$V_1(t)=\frac{\beta}{2}y_1^2+\frac{\beta}{2}z_1^2. \tag{12}$$

对$V_1(t)$求导, 随后代入式(11)得

$$\dot{V}_1(t)=\beta y_1(y_2+\tau_{1\mathrm{y}})+\beta z_1(z_2+\tau_{1\mathrm{z}}). \tag{13}$$

为了促使$\dot{V}_1(t)$负定, 设计虚拟控制为

$$\begin{cases}\tau_{1\mathrm{y}}=-\dfrac{\gamma}{\beta}y_1,\\ \tau_{1\mathrm{z}}=-\dfrac{\gamma}{\beta}z_1,\end{cases} \tag{14}$$

其中γ及β为两个正的常数. 将式(14)代入$\dot{V}_1(t)$中有

$$\dot{V}_1(t)=-\gamma y_1^2-\gamma z_1^2+\beta y_1y_2+\beta z_1z_2. \tag{15}$$

步骤 2 为了处理式(15)中的不定项, 修改Lyapunov候选函数为

$$V_2(t)=V_1(t)+\frac{\beta(m_{\mathrm{r}}+m_{\mathrm{b}})}{2}y_2^2+\frac{\beta m_{\mathrm{r}}}{2}z_2^2. \tag{16}$$

对上式求导得

$$\dot{V}_2(t)=\dot{V}_1(t)+\beta(m_{\mathrm{r}}+m_{\mathrm{b}})y_2\dot{y}_2+$$

$$\beta m_{\mathrm{r}} z_2 \dot{z}_2. \tag{17}$$

将式(8)–(9)(15)代入上式可得

$$\begin{aligned}\dot{V}_2(t) = &-\gamma y_1^2 - \gamma z_1^2 + \beta y_2[y_1 + \tau_{2\mathrm{y}} + \\ &y_3 + d_{\mathrm{q}}(t) - \mathrm{EI}u'''(0,t) - \\ &(m_{\mathrm{r}} + m_{\mathrm{b}})\dot{\tau}_{1\mathrm{y}}] + \beta z_2[z_1 + \\ &\tau_{2\mathrm{z}} + z_3 + d_{\mathrm{p}}(t) - \mathrm{EI}v'''(0,t) - \\ &m_{\mathrm{r}}\dot{\tau}_{1\mathrm{z}}].\end{aligned} \tag{18}$$

由反馈控制思想, 设计虚拟控制$\tau_{2\mathrm{y}}$和$\tau_{2\mathrm{z}}$如下:

$$\begin{aligned}\tau_{2\mathrm{y}} = &-k_1(x_{1\mathrm{y}} + \frac{\gamma x_{1\mathrm{y}}}{\beta}) + (m_{\mathrm{r}} + m_{\mathrm{b}})\dot{x}_{2\mathrm{y}} - \\ &x_{1\mathrm{y}} - M_1 \tanh(\frac{x_{1\mathrm{y}} + \frac{\gamma x_{1\mathrm{y}}}{\beta}}{c_{\mathrm{q}}}),\end{aligned} \tag{19}$$

$$\begin{aligned}\tau_{2\mathrm{z}} = &-k_2(x_{2z} + \frac{\gamma x_{1\mathrm{z}}}{\beta}) + m_{\mathrm{r}} x_{2\mathrm{z}} - \\ &x_{1\mathrm{z}} - M_2 \tanh(\frac{x_{1\mathrm{z}} + \frac{\gamma x_{1\mathrm{z}}}{\beta}}{c_{\mathrm{p}}}),\end{aligned} \tag{20}$$

其中$k_1, k_2, c_{\mathrm{p}}, c_{\mathrm{q}} > 0$.

将$\tau_{2\mathrm{y}}$和$\tau_{2\mathrm{z}}$代入式(18), 有

$$\begin{aligned}\dot{V}_2(t) = &-\gamma y_1^2 - k_1 y_2^2 - \beta \mathrm{EI} y_2 u'''(0,t) + \\ &\beta y_2[d_{\mathrm{q}}(t) - M_1 \tanh(\frac{y_2}{c_{\mathrm{q}}})] - \\ &\gamma z_1^2 - k_2 z_2^2 - \beta \mathrm{EI} z_2 v'''(0,t) + \\ &\beta z_2[d_{\mathrm{p}}(t) - M_2 \tanh(\frac{z_2}{c_{\mathrm{p}}})] + \\ &\beta(y_2 y_3 + z_2 z_3).\end{aligned} \tag{21}$$

由引理3可得

$$\beta y_2[d_{\mathrm{q}}(t) - M_1 \tanh(\frac{y_2}{c_{\mathrm{q}}})] \leqslant M_1 \beta \mu c_{\mathrm{q}}, \tag{22}$$

$$\beta z_2[d_{\mathrm{p}}(t) - M_2 \tanh(\frac{z_2}{c_{\mathrm{p}}})] \leqslant M_2 \beta \mu c_{\mathrm{p}}. \tag{23}$$

将式(22)–(23)代入式(21), 则$\dot{V}_2(t)$满足

$$\begin{aligned}&\dot{V}_2(t) \leqslant \\ &-\gamma y_1^2 - \gamma z_1^2 - k_1 y_2^2 - k_2 z_2^2 - \\ &\beta \mathrm{EI} y_2 u'''(0,t) - \beta \mathrm{EI} z_2 v'''(0,t) + \\ &\beta[y_2 y_3 + z_2 z_3 + M_1 \mu c_{\mathrm{q}} + M_2 \mu c_{\mathrm{p}}].\end{aligned} \tag{24}$$

步骤 3　为了镇定输入受限的核燃料装卸机系统, 进一步调整Lyapunov候选函数为

$$V_3(t) = V_2(t) + \frac{m_{\mathrm{r}} + m_{\mathrm{b}}}{2} y_3^2 + \frac{m_{\mathrm{r}}}{2} z_3^2. \tag{25}$$

为了抵消输入饱和的影响, 构造下列辅助系统:

$$\dot{v}_{\mathrm{q}}(t) = -b_{\mathrm{q}} v_{\mathrm{q}}(t) + \zeta_{\mathrm{q}}, \tag{26}$$

$$\dot{v}_{\mathrm{p}}(t) = -b_{\mathrm{p}} v_{\mathrm{p}}(t) + \zeta_{\mathrm{p}}, \tag{27}$$

其中: $b_{\mathrm{q}}, b_{\mathrm{p}} > 0$; ζ_{q}和ζ_{p}为待设计的辅助控制律.

对$V_3(t)$求导得

$$\begin{aligned}\dot{V}_3(t) = &\dot{V}_2(t) + y_3[(m_{\mathrm{r}} + m_{\mathrm{b}})\dot{\tau}_{2\mathrm{y}} - \\ &(m_{\mathrm{r}} + m_{\mathrm{b}})\dot{u}(v_{\mathrm{q}}(t))] + \\ &z_3[m_{\mathrm{r}}\dot{\tau}_{2\mathrm{z}} - m_{\mathrm{r}}\dot{u}(v_{\mathrm{p}}(t))].\end{aligned} \tag{28}$$

观察式 (19)–(20) 可知, 虚拟控制 $\tau_{2\mathrm{y}}$ 和 $\tau_{2\mathrm{z}}$ 由 $x_{1\mathrm{y}}$, $x_{2\mathrm{y}}$, $x_{1\mathrm{z}}$及$x_{2\mathrm{z}}$组成, 则

$$\begin{aligned}\dot{\tau}_{2\mathrm{y}} = &\frac{\partial \tau_{2\mathrm{y}}}{\partial x_{1\mathrm{y}}} x_{2\mathrm{y}} + \frac{1}{m_{\mathrm{r}} + m_{\mathrm{b}}} \frac{\partial \tau_{2\mathrm{y}}}{\partial x_{2\mathrm{y}}} (u(v_{\mathrm{q}}(t)) + \\ &d_{\mathrm{q}}(t) - \mathrm{EI}u'''(0,t)),\end{aligned} \tag{29}$$

$$\begin{aligned}\dot{\tau}_{2\mathrm{z}} = &\frac{\partial \tau_{2\mathrm{z}}}{\partial x_{1\mathrm{z}}} x_{2\mathrm{z}} + \frac{1}{m_{\mathrm{r}}} \frac{\partial \tau_{2\mathrm{z}}}{\partial x_{2\mathrm{z}}} (u(v_{\mathrm{p}}(t)) + \\ &d_{\mathrm{p}}(t) - \mathrm{EI}v'''(0,t)).\end{aligned} \tag{30}$$

分析式(26)–(27)得

$$\dot{u}(v_{\mathrm{q}}(t)) = \xi_{\mathrm{q}}(-b_{\mathrm{q}} v_{\mathrm{q}}(t) + \zeta_{\mathrm{q}}), \tag{31}$$

$$\dot{u}(v_{\mathrm{p}}(t)) = \xi_{\mathrm{p}}(-b_{\mathrm{p}} v_{\mathrm{p}}(t) + \zeta_{\mathrm{p}}), \tag{32}$$

其中ξ_{q}和ξ_{p}为

$$\xi_{\mathrm{q}} = \frac{\partial u(v_{\mathrm{q}}(t))}{\partial v_{\mathrm{q}}(t)} = \frac{4}{\mathrm{e}^{\frac{v_{\mathrm{q}}(t)}{u_1}} + \mathrm{e}^{\frac{-v_{\mathrm{q}}(t)}{u_1}}} > 0, \tag{33}$$

$$\xi_{\mathrm{p}} = \frac{\partial u(v_{\mathrm{p}}(t))}{\partial v_{\mathrm{p}}(t)} = \frac{4}{\mathrm{e}^{\frac{v_{\mathrm{p}}(t)}{u_2}} + \mathrm{e}^{\frac{-v_{\mathrm{p}}(t)}{u_2}}} > 0. \tag{34}$$

根据Nussbaum函数的性质, 设计辅助控制律ζ_{q}和ζ_{p}为

$$\zeta_{\mathrm{q}} = N(\chi_{\mathrm{q}})\bar{\zeta}_{\mathrm{q}}, \tag{35}$$

$$\zeta_{\mathrm{p}} = N(\chi_{\mathrm{p}})\bar{\zeta}_{\mathrm{p}}, \tag{36}$$

其中$N(\chi_{\mathrm{q}})$和$N(\chi_{\mathrm{p}})$为Nussbaum函数且被定义为

$$N(\chi_{\mathrm{q}}) = \chi_{\mathrm{q}}^2 \cos \chi_{\mathrm{q}},\ \dot{\chi}_{\mathrm{q}} = c_{\mathrm{q}}(m_{\mathrm{r}} + m_{\mathrm{b}}) y_3 \bar{\zeta}_{\mathrm{q}}, \tag{37}$$

$$N(\chi_{\mathrm{p}}) = \chi_{\mathrm{p}}^2 \cos \chi_{\mathrm{p}},\ \dot{\chi}_{\mathrm{p}} = c_{\mathrm{p}} m_{\mathrm{r}} z_3 \bar{\zeta}_{\mathrm{p}}, \tag{38}$$

其中$\bar{\zeta}_{\mathrm{q}}$及$\bar{\zeta}_{\mathrm{p}}$为

$$\begin{aligned}\bar{\zeta}_{\mathrm{q}} = &-\frac{1}{m_{\mathrm{r}} + m_{\mathrm{b}}}(k_3 y_3 - \frac{\partial \tau_{2\mathrm{y}}}{\partial x_{2\mathrm{y}}} u_{\mathrm{q}}(t) + \\ &\beta y_2 + \mathrm{EI}\frac{\partial \tau_{2\mathrm{y}}}{\partial x_{2\mathrm{y}}} u'''(0,t)) + \frac{\partial \tau_{2\mathrm{y}}}{\partial x_{1\mathrm{y}}} x_{2\mathrm{y}} + \\ &\xi_{\mathrm{q}} b_{\mathrm{q}} v_{\mathrm{q}}(t) - (\frac{\partial \tau_{2\mathrm{y}}}{\partial\, x_{2\mathrm{y}}})^2,\end{aligned} \tag{39}$$

$$\begin{aligned}\bar{\zeta}_{\mathrm{p}} = &\frac{1}{m_{\mathrm{r}}}(-k_4 z_3 - \frac{\partial \tau_{2\mathrm{z}}}{\partial x_{2\mathrm{z}}} u_{\mathrm{p}}(t) + \beta z_2 + \\ &\mathrm{EI}\frac{\partial \tau_{2\mathrm{z}}}{\partial x_{2\mathrm{z}}} v'''(0,t)) + \frac{\partial \tau_{2\mathrm{z}}}{\partial x_{1\mathrm{z}}} x_{2\mathrm{z}} + \\ &\xi_{\mathrm{p}} b_{\mathrm{p}} v_{\mathrm{p}}(t) - (\frac{\partial \tau_{2\mathrm{z}}}{\partial x_{2\mathrm{z}}})^2,\end{aligned} \tag{40}$$

其中$k_3, k_4 > 0$.

结合式(11)(39)–(40)可得

$$\dot{y}_3+\bar{\zeta}_{\rm q}=\xi_{\rm q}N(\chi_{\rm q})\bar{\zeta}_{\rm q}-(\frac{\partial\tau_{\rm 2y}}{\partial x_{\rm 2y}})^2y_3-\frac{1}{m_{\rm r}+m_{\rm b}}(k_3y_3+\beta y_2+\frac{\partial\tau_{\rm 2y}}{\partial x_{\rm 2y}}d_{\rm q}(t)),\tag{41}$$

$$\dot{z}_3+\bar{\zeta}_{\rm p}=\xi_{\rm p}N(\chi_{\rm p})\bar{\zeta}_{\rm p}-(\frac{\partial\tau_{\rm 2z}}{\partial x_{\rm 2z}})^2z_3-\frac{1}{m_{\rm r}}(k_4z_3+\beta z_2+\frac{\partial\tau_{\rm 2z}}{\partial x_{\rm 2z}}d_{\rm p}(t)).\tag{42}$$

运用引理1及假设1推导出下列不等式:

$$\frac{\partial\tau_{\rm 2y}}{\partial x_{\rm 2y}}y_3d_{\rm q}(t)\leqslant(m_{\rm r}+m_{\rm b})(\frac{\partial\tau_{\rm 2y}}{\partial x_{\rm 2y}}y_3)^2+\frac{M_1^2}{m_{\rm r}+m_{\rm b}}.\tag{43}$$

针对$\frac{\partial\tau_{\rm 2z}}{\partial x_{\rm 2z}}z_3d_{\rm p}(t)$, 同理有

$$\frac{\partial\tau_{\rm 2z}}{\partial x_{\rm 2z}}z_3d_{\rm p}(t)\leqslant m_{\rm r}(\frac{\partial\tau_{\rm 2z}}{\partial x_{\rm 2z}}z_3)^2+\frac{M_2^2}{m_{\rm r}}.\tag{44}$$

结合式(28)(41)–(44)得

$$\begin{aligned}\dot{V}_3(t)\leqslant&-\gamma y_1^2-\gamma z_1^2-k_1y_2^2-k_2z_2^2-\\&k_3y_3^2-k_4z_3^2-\beta{\rm EI}[y_2u'''(0,t)-\\&z_2v'''(0,t)]+\frac{1}{c_{\rm q}}[\xi_{\rm q}N(\chi_{\rm q})-1]\dot{\chi}_{\rm q}+\\&\frac{1}{c_{\rm p}}[\xi_{\rm z}N(\chi_{\rm p})-1]\dot{\chi}_{\rm p}+\frac{M_1^2}{m_{\rm r}+m_{\rm b}}+\\&\frac{M_2^2}{m_{\rm r}}+M_1\beta\mu c_{\rm q}+M_2\beta\mu c_{\rm p}.\end{aligned}\tag{45}$$

4 一致有界稳定

为了分析系统的稳定性, 构建最终的Lyapunov函数如下:

$$V(t)=V_{\rm e}(t)+V_{\rm c}(t)+V_3(t),\tag{46}$$

其中$V_{\rm e}(t)$和$V_{\rm c}(t)$为

$$\begin{aligned}V_{\rm e}(t)=&\frac{\beta\rho}{2}\int_0^l[\dot{u}(x,t)+\dot{q}(t)]^2{\rm d}x+\\&\frac{\beta\rho}{2}\int_0^l[\dot{v}(x,t)+\dot{p}(t)]^2{\rm d}x+\\&\frac{\beta{\rm EI}}{2}\int_0^l[(u''(x,t))^2+(v''(x,t))^2]{\rm d}x,\end{aligned}\tag{47}$$

$$\begin{aligned}V_{\rm c}(t)=&\gamma\rho\int_0^l[u(x,t)+y_1][\dot{u}(x,t)+\dot{q}(t)]{\rm d}x+\\&\gamma\rho\int_0^l[v(x,t)+z_1][\dot{v}(x,t)+\dot{p}(t)]{\rm d}x.\end{aligned}\tag{48}$$

由引理1及2可得

$$\begin{aligned}|V_{\rm c}(t)|\leqslant&\frac{\gamma l\rho}{2}y_1^2+\frac{\gamma\rho l^3}{2}\int_0^l[u''(x,t)]^2{\rm d}x+\\&\frac{\gamma l\rho}{2}z_1^2+\frac{\gamma\rho l^3}{2}\int_0^l[v''(x,t)]^2{\rm d}x+\\&\frac{\gamma\rho}{2}\int_0^l[\dot{u}(x,t)+\dot{q}(t)]^2{\rm d}x+\\&\frac{\gamma\rho}{2}\int_0^1[\dot{v}(x,t)+\dot{p}(t)]^2{\rm d}x.\end{aligned}\tag{49}$$

选择合适的γ满足$0<\gamma\leqslant\beta\min\{\frac{1}{2},\frac{\rm EI}{\rho l^3},\frac{1}{\rho l}\}$, 则下列不等式成立:

$$|V_{\rm c}(t)|\leqslant\lambda_0[V_{\rm e}(t)+V_3(t)],\tag{50}$$

其中$\lambda_0=\frac{\gamma}{\beta}\max\{1,\frac{\rho l^3}{\rm EI},\rho l\}$.

结合式(49)–(50)得

$$\begin{aligned}0<\lambda_1[V_{\rm e}(t)+V_3(t)]\leqslant V(t)\leqslant\\\lambda_2[V_{\rm e}(t)+V_3(t)],\end{aligned}\tag{51}$$

其中: $\lambda_1=\min\{1-\frac{\gamma}{\beta},1-\frac{\gamma\rho l^3}{\beta{\rm EI}},1-\frac{\gamma\rho l}{\beta}\}>0$, $\lambda_2=\max\{1+\frac{\gamma}{\beta},1+\frac{\gamma\rho l^3}{\beta{\rm EI}},\frac{\gamma\rho l}{\beta}\}>0$.

综上分析, 由式(51)可推出$V(t)$正定.

引理 5 存在两个正的常数η及ω, 使得$V(t)$满足下列不等式:

$$\dot{V}(t)\leqslant-\eta V(t)+\omega+\frac{1}{c_{\rm q}}[\xi_{\rm q}N(\chi_{\rm q})-1]\dot{\chi}_{\rm q}+\frac{1}{c_{\rm p}}[\xi_{\rm p}N(\chi_{\rm p})-1]\dot{\chi}_{\rm p}.\tag{52}$$

证 对$V(t)$求导可得

$$\dot{V}(t)=\dot{V}_{\rm e}(t)+\dot{V}_{\rm c}(t)+\dot{V}_3(t).\tag{53}$$

对式(47)求导并运用引理1和假设1可导出

$$\begin{aligned}\dot{V}_{\rm e}(t)\leqslant&-(\beta c-\delta_1)\int_0^l[\dot{u}(x,t)+\dot{q}(t)]^2{\rm d}x-\\&(\beta c-\delta_2)\int_0^l[\dot{v}(x,t)+\dot{p}(t)]^2{\rm d}x+\\&\beta{\rm EI}\dot{q}(t)u'''(0,t)+\frac{l}{\delta_1}F_1^2+\\&\beta{\rm EI}\dot{p}(t)v'''(0,t)+\frac{l}{\delta_2}F_2^2.\end{aligned}\tag{54}$$

对式(48)求导, 随后代入式(4)–(5), 运用引理1和假设1可知

$$\begin{aligned}\dot{V}_{\rm c}(t)\leqslant&\frac{\gamma c}{\delta_4}\int_0^l[\dot{u}(x,t)+\dot{q}(t)]^2{\rm d}x+\\&\frac{\gamma c}{\delta_6}\int_0^l[\dot{v}(x,t)+\dot{p}(t)]^2{\rm d}x-\\&\int_0^l[I_1(u''(x,t))^2+I_2(v''(x,t))^2]{\rm d}x+\\&(2\gamma\delta_3l+2\gamma\delta_4cl)y_1^2+\gamma{\rm EI}y_1u'''(0,t)+\\&(2\gamma\delta_5l+2\gamma\delta_6cl)z_1^2+\gamma{\rm EI}z_1v'''(0,t)+\\&\frac{\gamma l}{\delta_3}F_1^2+\frac{\gamma l}{\delta_5}F_2^2,\end{aligned}\tag{55}$$

其中: $I_1=\gamma\mathrm{EI}-2\gamma\delta_3 l^3-2\gamma\delta_4 cl^3$, $I_2=\gamma\mathrm{EI}-2\gamma\delta_5 l^3-2\gamma\delta_6 cl^3$.

将式(45)(54)–(55)代入式(53)可得

$$\begin{aligned}\dot V(t)\leqslant&-I_3\int_0^l(\dot u(x,t)+\dot q(t))^2\mathrm{d}x-\\&I_4\int_0^l(\dot v(x,t)+\dot p(t))^2\mathrm{d}x-\\&\int_0^l[I_1(u''(x,t))^2+I_2(v''(x,t))^2]\mathrm{d}x-\\&I_5y_1^2-k_1y_2^2-k_3y_3^2-I_6z_1^2-\\&k_2z_2^2-k_4z_3^2+\frac{1}{c_\mathrm{q}}[\xi_\mathrm{q}N(\chi_\mathrm{q})-1]\dot\chi_\mathrm{q}+\\&\frac{1}{c_\mathrm{p}}[\xi_\mathrm{p}N(\chi_\mathrm{p})-1]\dot\chi_\mathrm{z}+\omega,\end{aligned}\tag{56}$$

其中:

$I_3=\beta c-\beta\delta_1-\dfrac{\gamma c}{\delta_4}$, $I_4=\beta c-\beta\delta_2-\dfrac{\gamma c}{\delta_6}$,

$I_5=\gamma-2\gamma\delta_3 l-2\gamma\delta_4 cl$, $I_6=\gamma-2\gamma\delta_5 l-2\gamma\delta_6 cl$,

$$\begin{aligned}\omega=&\frac{M_1^2}{m_\mathrm{r}+m_\mathrm{b}}+\frac{M_2^2}{m_\mathrm{r}}+M_1\beta\mu c_\mathrm{q}+M_2\beta\mu c_\mathrm{p}+\\&(\frac{l}{\delta_1}+\frac{\gamma l}{\delta_3})F_1^2+(\frac{l}{\delta_2}+\frac{\gamma l}{\delta_5})F_2^2>0.\end{aligned}$$

选择合适的γ,β及$\delta_i(i=1,\cdots,6)$以满足下列条件:

$$\begin{cases}\beta\leqslant\gamma c\min\{\dfrac{1}{\delta_4(\delta_1-c)},\ \dfrac{1}{(\delta_2-c)\delta_6}\},\\ \delta_3+\delta_4\leqslant\dfrac{\mathrm{EI}}{2l^3(1+c)},\\ \delta_5+\delta_6\leqslant\dfrac{\mathrm{EI}}{2l(1+c)},\\ \gamma\geqslant\max\{\dfrac{1}{2\delta_3 l+2\delta_4 cl},\ \dfrac{1}{2\delta_5 l^3+2\delta_6 cl}\}.\end{cases}\tag{57}$$

结合式(51)(56)得

$$\begin{aligned}\dot V(t)\leqslant&-\eta V(t)+\omega+\frac{1}{c_\mathrm{q}}[\xi_\mathrm{q}N(\chi_\mathrm{q})-1]\dot\chi_\mathrm{q}+\\&\frac{1}{c_\mathrm{p}}[\xi_\mathrm{p}N(\chi_\mathrm{p})-1]\dot\chi_\mathrm{p},\end{aligned}\tag{58}$$

其中$\eta=\dfrac{\lambda_3}{\lambda_2}>0$, λ_3为

$$\begin{aligned}\lambda_3=&\frac{2}{\beta}\min\{\frac{I_3}{\rho},\frac{I_4}{\rho},\frac{I_1}{\mathrm{EI}},\frac{I_2}{\mathrm{EI}},I_5,I_6,\\&\frac{k_1}{m_\mathrm{b}+m_\mathrm{r}},\frac{k_2}{m_\mathrm{r}},\frac{\beta k_3}{m_\mathrm{b}+m_\mathrm{r}},\frac{\beta k_4}{m_\mathrm{r}}\}>0.\end{aligned}\tag{59}$$

证毕.

定理 1　对式(4)–(9)所描述的核燃料装卸机系统, 式(10)所给出的边界控制方案可以确保闭环系统一致有界稳定.

证　由引理5可得

$$\begin{aligned}V(t)\leqslant&V(0)\mathrm{e}^{-\eta t}+\frac{1}{\eta}\omega(1-\mathrm{e}^{-\eta t})+\\&\frac{\mathrm{e}^{-\eta t}}{c_\mathrm{q}}\int_0^t(\xi_\mathrm{q}N(\chi_\mathrm{q})-1)\dot\chi_\mathrm{q}\mathrm{e}^{-\eta\tau}\mathrm{d}\tau+\\&\frac{\mathrm{e}^{-\eta t}}{c_\mathrm{p}}\int_0^t(\xi_\mathrm{p}N(\chi_\mathrm{p})-1)\dot\chi_\mathrm{p}\mathrm{e}^{-\eta\tau}\mathrm{d}\tau.\end{aligned}\tag{60}$$

根据引理4可知$V(t),\chi_\mathrm{q},\chi_\mathrm{p},\int_0^t(\xi_\mathrm{q}N(\chi_\mathrm{q})-1)\times\dot\chi_\mathrm{q}\mathrm{e}^{-\eta\tau}\mathrm{d}\tau$和$\int_0^t(\xi_\mathrm{p}N(\chi_\mathrm{p})-1)\dot\chi_\mathrm{p}\mathrm{e}^{-\eta\tau}\mathrm{d}\tau$一致有界.

运用引理2可以得到

$$\begin{cases}\dfrac{\beta\mathrm{EI}}{2l^3}u^2(x,t)\leqslant\dfrac{\beta\mathrm{EI}}{2}\displaystyle\int_0^l[u''(x,t)]^2\mathrm{d}x\leqslant\dfrac{V(t)}{\lambda_1},\\ \dfrac{\beta\mathrm{EI}}{2l^3}v^2(x,t)\leqslant\dfrac{\beta\mathrm{EI}}{2}\displaystyle\int_0^l[v''(x,t)]^2\mathrm{d}x\leqslant\dfrac{V(t)}{\lambda_1},\\ \dfrac{1}{2}y_1^2\leqslant V_3(t)\leqslant\dfrac{V(t)}{\lambda_1},\\ \dfrac{1}{2}z_1^2\leqslant V_3(t)\leqslant\dfrac{V(t)}{\lambda_1}.\end{cases}\tag{61}$$

将式(60)代入式(61)可以推出

$$\begin{cases}|u(x,t)|\leqslant\sqrt{\dfrac{2l^3}{\beta\mathrm{EI}\lambda_1}[V(0)+\dfrac{\phi}{\eta}]},\\ |v(x,t)|\leqslant\sqrt{\dfrac{2l^3}{\beta\mathrm{EI}\lambda_1}[V(0)+\dfrac{\phi}{\eta}]},\\ |y_1|\leqslant\sqrt{\dfrac{2}{\lambda_1}[V(0)+\dfrac{\phi}{\eta}]},\\ |z_1|\leqslant\sqrt{\dfrac{2}{\lambda_1}[V(0)+\dfrac{\phi}{\eta}]},\end{cases}\tag{62}$$

其中$\phi=\omega+\int_0^t(\xi_\mathrm{q}N(\chi_\mathrm{q})-1)\dot\chi_\mathrm{q}\mathrm{e}^{-\eta\tau}\mathrm{d}\tau+\int_0^t(\xi_\mathrm{p}\times N(\chi_\mathrm{p})-1)\dot\chi_\mathrm{p}\mathrm{e}^{-\eta\tau}\mathrm{d}\tau$.　证毕.

5　数字仿真

为了验证本文所设计边界控制方案的有效性, 采用有限差分法拟合核燃料装卸机系统的动力学响应. 选取时间步长和空间步长分别为0.001和0.025, 系统物理结构参数为: $l=1$ m, $d=0.02$ m, $\mathrm{EI}=100\ \mathrm{Nm}^2$, $m_\mathrm{r}=5$ kg, $m_\mathrm{b}=7$ kg, $\rho=2\,000$ kg/m, $c=0.2$ Ns/m. 考虑外部干扰为$F_\mathrm{p}(x,t)=(1+x)\times\sin(0.8t)$, $F_\mathrm{q}(x,t)=2x(\cos(0.2t)+1.5\sin(0.1t))$, $d_\mathrm{p}(t)=\sin(0.1\pi t)$及$d_\mathrm{q}(t)=\sin(0.2\pi t)$. 设定系统初始状态为$u(x,0)=0.1x$, $v(x,0)=0.1x^2$, 初始速度为$\dot u(x,0)=0$, $\dot v(x,0)=0$.

当选择控制参数为$b_\mathrm{p}=1.5$, $b_\mathrm{q}=2$, $k_1=10$, $k_2=$

$20, k_3 = 20, k_4 = 40$, 图2–9给出了仿真结果, 其中; 图2–3为无控制作用时燃料棒的侧向和横向振动偏移量, 图4–5展示了在本文所设计的边界控制作用下的燃料棒的侧向和横向振动偏移量, 图6–7给出了导轨和运输小车的位置跟踪图, 图8–9为边界控制输入. 由上述仿真结果可得出如下结论:

1) 观察图2–3得, 无控制时燃料棒不稳定, 燃料棒始终在振动且其振动偏移量较大;

2) 比较图2–3与图4–5知, 在外部干扰的影响下, 本文设计的边界控制能大幅度抵消水下移动的燃料棒的振动, 并且燃料棒的振动位移在4 s后稳定在零点附近;

3) 分析图6–7可得, 提出的控制方案可以促使导轨及运输小车分别到达指定位置0.6 m和0.2 m处附近, 即, 带有边界控制的装卸机可以将燃料棒运送至期望位置附近;

4) 由图8–9可知, 尽管边界控制输入$u_{\rm q}(t)$和$u_{\rm p}(t)$的变动范围限制在$-20\sim20$ N和$-40\sim40$ N, 本文设计的控制方案依然可以实现预期的控制目标.

综合分析上述结论可知, 对于受外部干扰影响且带输入饱和的核燃料装卸机, 本文所设计的边界控制具有较好的振动抑制及位置跟踪能力.

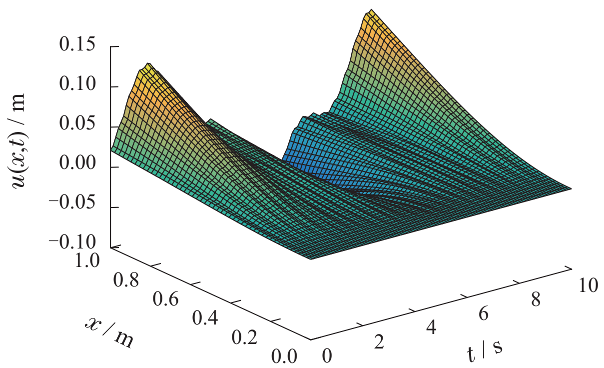

图 2 无控制时燃料棒侧向振动位移

Fig. 2 Lateral vibration displacement of fuel rod without control

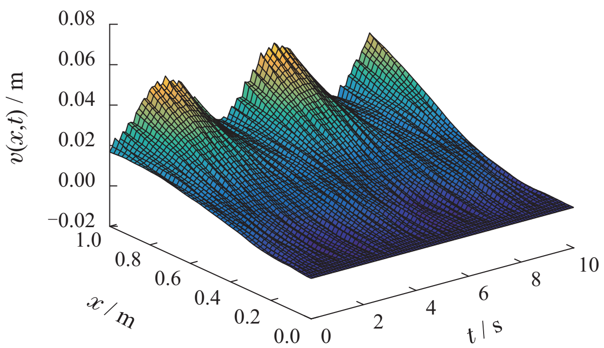

图 3 无控制时燃料棒横向振动位移

Fig. 3 Transverse vibration displacement of fuel rod without control

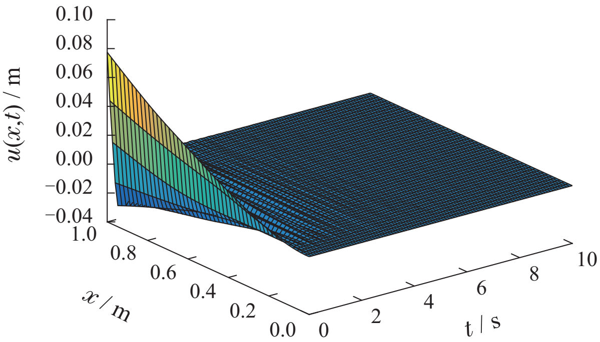

图 4 边界控制时燃料棒侧向振动位移

Fig. 4 Lateral vibration displacement of fuel rod with boundary control

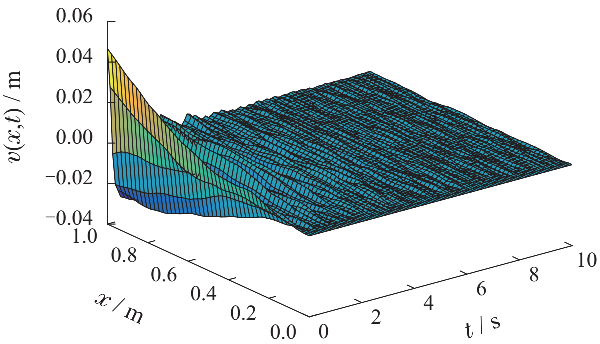

图 5 边界控制时燃料棒横向振动位移

Fig. 5 Transverse vibration displacement of fuel rod with boundary control

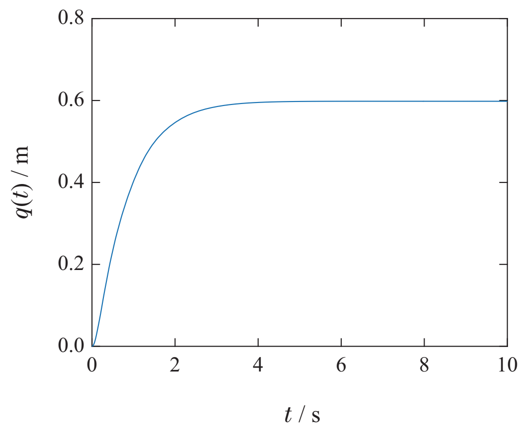

图 6 边界控制时导轨位置

Fig. 6 Position of guide rail with boundary control

6 结论

本文解决了受外部干扰及输入饱和影响的核燃料装卸机的振动抑制及位置跟踪问题. 通过分析核燃料装卸机系统的能量及采用Hamilton原理, 建立了精确的分布参数模型. 运用反步法设计了边界控制方案来抵消燃料棒的振动并将其运送至给定位置. 构造的辅助系统及Nussbaum函数处理了输入饱和的影响. 引进了双曲正切函数来抵消外部干扰. 经Lyapunov稳定性分析可知所提的边界控制可以一致有界地镇定核燃

料装卸机系统. 仿真结果表明所给出的边界控制方案具有良好的控制效果.

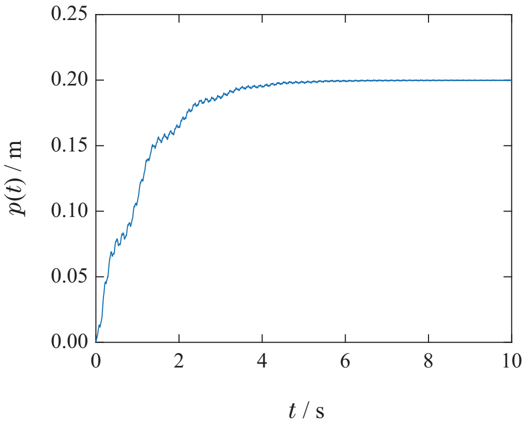

图 7　边界控制时小车位置

Fig. 7　Position of machine with boundary control

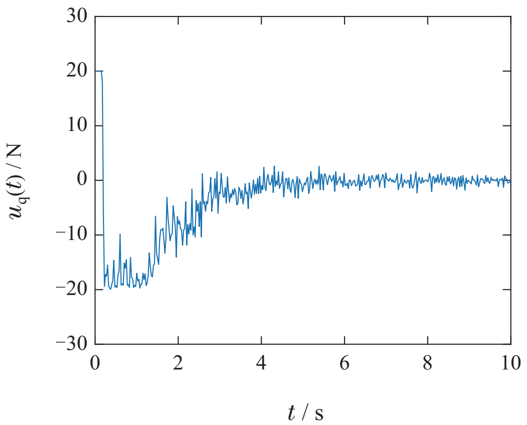

图 8　控制输入$u_{\rm q}(t)$

Fig. 8　Control input $u_{\rm q}(t)$

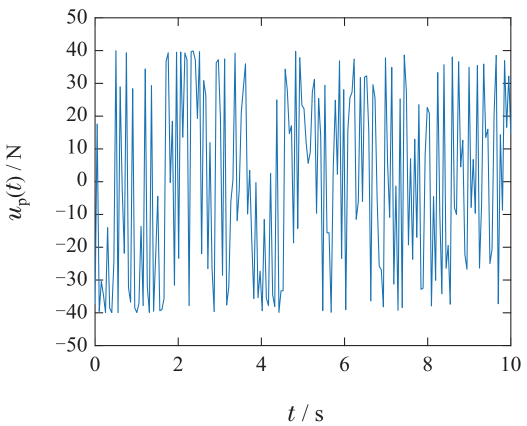

图 9　控制输入$u_{\rm p}(t)$

Fig. 9　Control input $u_{\rm p}(t)$

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